# Section 4.10: Antiderivatives

Given a function , how do we find a function with derivative and why would we be interested in such a function?

## The Reverse of Differentiation

A function is an **antiderivative** of the function if

for all in the domain of .

Knowing the rules of differentiation helps begin to find the antiderivatives of functions. For instance, consider the function . An antiderivative of could be since . However, that is not the only function that has a derivative of . Since the derivative of any constant is zero, other functions, such as or could also have a derivative of .

**General Form of an Antiderivative**

Let be an antiderivative of over an interval . Then,

1. for each constant , the function is also an antiderivative of over ;
2. if is an antiderivative of over , there is a constant for which over .

In other words, the most general form of an antiderivative of over is .

Media: Watch this [video](https://youtu.be/b-fniNh2Aug) example on antiderivatives of monomials.

Media: Watch this [video](https://youtu.be/KM61G41NKGY) example on antiderivatives of rational functions.

Media: Watch this [video](https://youtu.be/o-mjR1OKOik) example on antiderivatives of trigonometric functions.

Examples: For each of the following functions, find all antiderivatives.

## Indefinite Integrals

We now look at the formal notation used to represent antiderivatives and examine some of their properties. Recall that when given a function , we use the notation or to denote the derivative of .

Given a function , the **indefinite integral** of , denoted

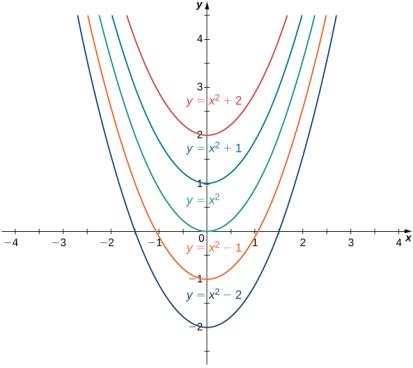
,

Is the most general antiderivative of . If is an antiderivative of , then

.

The expression is called the **integrand** and the variable is the **variable of integration**. The act of finding the antiderivatives of a function is usually referred to as **integrating** .

For a function and an antiderivative , the functions , where is any real number, are often referred to as the **family of antiderivatives** of . For example, the collection of all functions of the form , where is any real number, is known as the family of antiderivatives of . The figure below shows a graph of this family.



The following table lists the indefinite integrals for several common functions.

| Differentiation Formula | Indefinite Integral |
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We can evaluate indefinite integrals for more complicated functions by using different properties of indefinite integrals.

**Properties of Indefinite Integrals**

Let and be antiderivatives of and , respectively, and let be any real number.

**Sums and Differences**

**Constant Multiplies**

Examples: Evaluate each of the following indefinite integrals.

## Initial Value Problems

One common use for antiderivatives that arises often in many applications is solving differential equations. A **differential equation** is an equation that relates an unknown function and one or more of its derivatives.

Sometimes we are interested in determining whether a particular solution curve passes through a certain point – that is, (the initial condition). The problem of finding a function that satisfies a differential equation

With the additional condition

Is an example of an **initial-value problem**.

Media: Watch this [video](https://youtu.be/bvlIWYMDqNk) example on solving initial value problems.

Examples

1. Solve the initial-value problem .
2. A car is traveling at the rate of () when the brakes are applied. The car begins decelerating at a constant rate of .
   1. How many seconds elapse before the car stops?
   2. How far does the car travel during that time?